Math 7

Mod 3 Topic 1

No Substitute for Hard Work

1

Evaluating Algebraic Expressions

WARM UP

Perform each operation.

1. $(-3)(6.6)$

2. $-3 + 6.6$

3. $-3 - 6.6$

4. $6.6 \div (-3)$

LEARNING GOALS

- Compare unknown quantities on a number line.
- Define linear expressions.
- Evaluate algebraic expressions.
- Solve real-life and mathematical problems using algebraic expressions.

KEY TERMS

- variable
- algebraic expression
- linear expression
- constraint
- evaluate an algebraic expression

You have written and evaluated algebraic expressions with positive rational numbers. How do you evaluate algebraic expressions over the set of rational numbers?

Getting Started

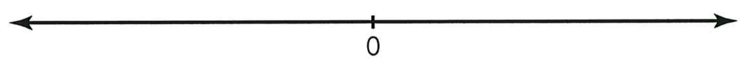
In algebra, a **variable** is a letter or symbol that is used to represent an unknown quantity.

The Empty Number Line

Consider the list of six *variable* expressions:

x $2x$ $3x$ $\frac{1}{2}x$ $-x$ $-\frac{1}{2}x$

1. With your partner, think about where you would place each expression and sketch your conjecture.



2. Compare your number line with another group's number line. What is the same? What is different?

3. Your teacher will select students to place an index card representing each expression on the number line on the board. Record the locations agreed upon by the class.

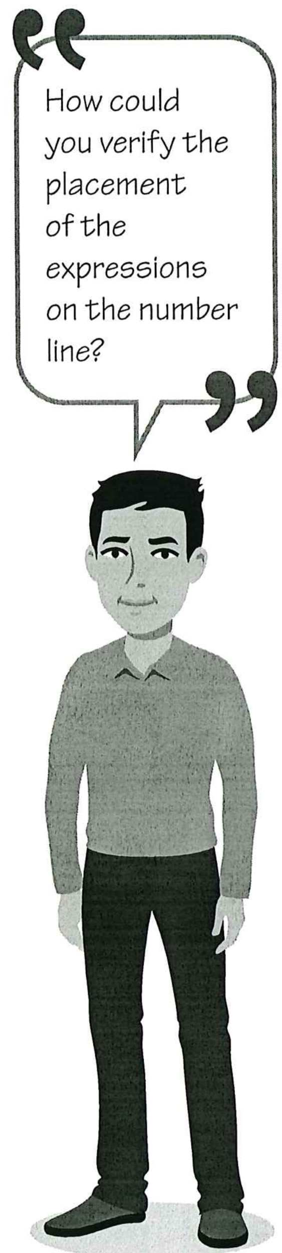
ACTIVITY
1.1 Algebraic Expressions

In this lesson, you will explore the relationship between unknown quantities by writing and evaluating *algebraic expressions*. An **algebraic expression** is a mathematical phrase that has at least one variable, and it can contain numbers and operation symbols.

Each of the expressions in the *Empty Number Line* activity is an algebraic expression. They are also *linear expressions*. A **linear expression** is any expression in which each term is either a constant or the product of a constant and a single variable raised to the first power. * so no exponents (exponents of 1 are not written)

Additional examples of linear expressions include:
$\frac{1}{2}x + 2$, $-3 + 12.5x$, $-1 + 3x + \frac{5}{2}x - \frac{4}{3}$, or $4y$.

The expressions $3x^2 + 5$ and $-\frac{1}{2}xy$ are examples of expressions that are not linear expressions.



1. **Provide a reason why each expression does not represent a linear expression.**

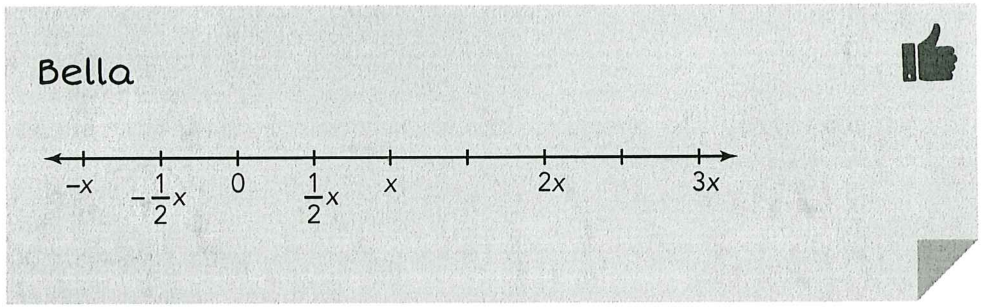
$3x^2 + 5$ is not linear because of the x^2.

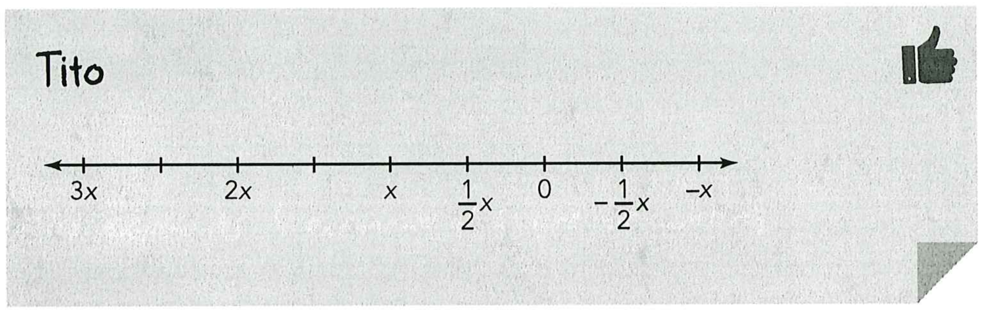
$-\frac{1}{2}xy$ is not linear because it has a product of 2 variables. (xy)

Let's revisit how you may have plotted the expressions in the previous activity. The directions did not specify the possible values for x. When you graphed each expression, did you think about the set of all possible values of x or just the set of positive x-values?

In mathematics, it is sometimes necessary to set *constraints* on values. A **constraint** is a condition that a solution or problem must satisfy. A constraint can be a restriction set in advance of solving a problem or a limit placed on a solution or graph so the answer makes sense in terms of a real-world scenario.

Analyze the number lines created by Bella and Tito using the expressions from the *Empty Number Line* activity.





2. Compare and contrast each representation.

a. What are the constraints on each representation? Identify the set of x-values that make each number line true.

For Bella's representation $x > 0$ (its to the right of 0)

For Tito's representation $x < 0$ (its to the left of 0)

b. Select a value for x from your set of possible values and substitute that value for x in each expression to verify the plotted locations are correct.

Bella $x = 4$ $2x = 8$ $3x = 12$ $\frac{1}{2}x = 2$ $-\frac{1}{2}x = -2$ $-x = -4$

Tito $x = -2$ $2x = -4$ $3x = -6$ $\frac{1}{2}x = -1$ $-\frac{1}{2}x = 1$ $-x = 2$

c. Compare your values from part (b) with your classmates. Do you have the same values? If not, what does that mean?

One strategy to verify your placement of the cards is to substitute values for the variable x into each expression.

ACTIVITY 1.2

Substitution with Rational Numbers

To earn money for a summer mission trip, Levi is working as a handyman around his neighborhood. Levi has been hired to build a wooden fence. He plans to use a post hole digger to dig the holes for the posts.

Levi starts the project on Saturday morning but because of the type of soil, he only starts the holes, fills them with water, and then plans to return Sunday to finish the job. When Levi starts on Sunday, each hole is 3 inches deep. Each time he uses the post hole diggers, he extracts 2 inches of soil. The height of the soil in the hole with respect to ground level can be modeled by the linear expression $-3 - 2n$, where n is the number of times Levi extracted soil with the post hole diggers.

A depth of 3 inches is equivalent to a height of -3 inches.

1. Determine the height of the soil in the hole as Levi works.

n $\quad$ $-3-2n$

Number of Soil Extractions	Height of the Soil (inches)	
0	-3	$-3-2(0) = -3-0 = -3$
1	-5	$-3-2(1) = -3-2 = -5$
2	-7	$-3-2(2) = -3-4 = -7$
5	-13	$-3-2(5) = -3-10 = -13$
10	-23	$-3-2(10) = -3-20 = -23$
15	-33	$-3-2(15) = -3-30 = -33$
20	-43	$-3-2(20) = -3-40 = -43$

* ↑ multiply this # by 2 & subtract it from -3 to get #

Use the Order of Operations to evaluate the expressions.

2. From his research about digging post holes, Levi knows that each pole must be placed at a depth that is 2 feet below the frost level, and the frost level is 16 inches beneath ground level.

   a. How deep must Levi dig each hole?

   2 ft = 24 in

   $-16 + -24 = \boxed{-40 \text{ in}}$

   b. Determine the minimum number of soil extractions for each hole.

   $-40 = -3 - 2n$
   $+3 \quad +3$
   $\frac{-37}{-2} = \frac{-2n}{-2}$

   $n = 18.5$

   $\boxed{19 \text{ extractions}}$

Cable drilling, also known as percussion drilling, is a method used to drill a borehole.

Levi's mom, Maggie, uses a cable tool rig to dig wells during the mission trip. Her rig can dig 12.4 meters of hard rock per day. When Maggie starts working on one well, the hole is already 33 meters deep.

3. Write a linear expression for the height of the hole with respect to ground level for the number of days that Maggie runs the rig.

   $-33 - 12.4d$ d = # of days

4. Use your expression to determine the height of the hole after each number of days.

   a. 2 days after Maggie starts

   $d = 2$ $-33 - 12.4(2) = -33 - 24.8 = \boxed{-57.8 \text{ m}}$

   b. 5 days after Maggie starts

   $d = 5$ $-33 - 12.4(5) = -33 - 62 = \boxed{-95 \text{ m}}$

   c. 2 days before Maggie started

   $-33 \le h \le 0$ You can't tell for sure. It could have been at 0 (ground level) or any depth up to -33 in.

ACTIVITY 1.3

Evaluating Expressions

Previously, you evaluated algebraic expressions with positive rational numbers. Now you can evaluate expressions with negative rational numbers. To **evaluate an algebraic expression**, you replace each variable in the expression with a number or numeric expression and then perform all possible mathematical operations.

1. **Evaluate each algebraic expression.**

a. $x - 7$

- for $x = -8$ $-8-7=-15$
- for $x = -11$ $-11-7=-18$
- for $x = 16$ $16-7=9$

b. $-6y$

- for $y = -3$ $-6(-3)=18$
- for $y = 0$ $-6(0)=0$
- for $y = 7$ $-6(7)=-42$

Use parentheses to show multiplication like $-6(-3)$.

c. $3b - 5$

① • for $b = -2$ -11
② • for $b = 3$ 4
③ • for $b = 9$ 22

① $3(-2)-5$
$-6-5$
-11

② $3(3)-5$
$9-5$
4

③ $3(9)-5$
$27-5$
22

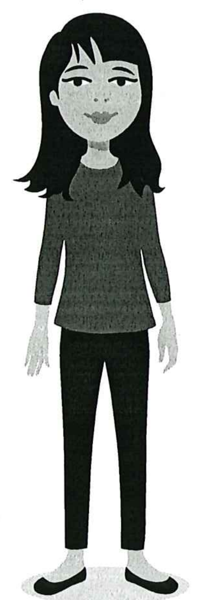
d. $-1.6 + 5.3n$

① • for $n = -5$ -28.1
② • for $n = 0$ -1.6
③ • for $n = 4$ 19.6

① $-1.6+5.3(-5)$
$-1.6-26.5$
-28.1

② $-1.6+5.3(0)$
$-1.6+0$
-1.6

③ $-1.6+5.3(4)$
$-1.6+21.2$
19.6



Sometimes, it is more convenient to use a table to record the results when evaluating the same expression with multiple values.

2. Complete each table.

a.

h	$-2h - 7$
2	-11
-1	-5
8	-23
-7	7

$-2(2) - 7$
$-4 - 7 = -11$
$-2(-1) - 7$
$2 - 7 = -5$
$-2(8) - 7$
$-16 - 7 = -23$
$-2(-7) - 7$
$14 - 7 = 7$

b.

a	-12	-10	-4	0
$\frac{a}{4} + 6$	3	3.5	5	6

$\frac{-12}{4} + 6$ $\frac{-10}{4} + 6$ $\frac{-4}{4} + 6$ $\frac{0}{4} + 6$

c.

x	$x^2 - 5$
1	-4
3	4
6	31
-2	-1

$(1)^2 - 5$
$1 - 5 = -4$
$(3)^2 - 5$
$9 - 5 = 4$
$(6)^2 - 5$
$36 - 5 = 31$
$(-2)^2 - 5$
$4 - 5 = -1$

d.

y	-5	-1	0	15
$-\frac{1}{5}y + 3\frac{2}{5}$	$4\frac{2}{5}$	$3\frac{3}{5}$	$3\frac{2}{5}$	$\frac{2}{5}$

$-\frac{1}{5}(-5) + 3\frac{2}{5}$
$1 + 3\frac{2}{5}$
$4\frac{2}{5}$

$-\frac{1}{5}(-1) + 3\frac{2}{5}$
$\frac{1}{5} + 3\frac{2}{5}$
$3\frac{3}{5}$

$-\frac{1}{5}(0) + 3\frac{2}{5}$
$0 + 3\frac{2}{5}$
$3\frac{2}{5}$

$-\frac{1}{5}(15) + 3\frac{2}{5}$
$-3 + 3\frac{2}{5}$
$\frac{2}{5}$

3. Evaluate each algebraic expression for $x = 2, -3, 0.5$, and $-2\frac{1}{3}$.

a. $-3x$

$-3(2) = -6$

$-3(-3) = 9$

$-3(0.5) = -1.5$

$-3(-2\frac{1}{3}) = 7$

b. $5x + 10$

$5(2)+10 = 10+10 = 20$

$5(-3)+10 = -15+10 = -5$

$5(0.5)+10 = 2.5+10 = 12.5$

$5(-2\frac{1}{3})+10 = -11\frac{2}{3}+10 = -1\frac{2}{3}$

c. $6 - 3x$

$6-3(2) = 6-6 = 0$

$6-3(-3) = 6+9 = 15$

$6-3(0.5) = 6-1.5 = 4.5$

$6-3(-2\frac{1}{3}) = 6+7 = 13$

d. $8x + 75$

$8(2)+75 = 16+75 = 91$

$8(-3)+75 = -24+75 = 51$

$8(0.5)+75 = 4+75 = 79$

$8(-2\frac{1}{3})+75 = -18\frac{2}{3}+75 = 56\frac{1}{3}$

4. Evaluate each algebraic expression for $x = 23.76$ and $-21\frac{5}{6}$.

a. $2.67x - 31.85$

$2.67(23.76)-31.85$

$63.4392-31.85$

$\boxed{31.5892}$

$2.67(-21\frac{5}{6})-31.85$

$-58.295-31.85$

$\boxed{-90.145}$

b. $11\frac{3}{4}x + 56\frac{3}{8}$

$11\frac{3}{4}(23.76)+56\frac{3}{8}$

$279.18+56\frac{3}{8}$

$\boxed{335.555}$

$11\frac{3}{4}(-21\frac{5}{6})+56\frac{3}{8}$

$-256\frac{13}{24}+56\frac{3}{8}$

$\boxed{-200\frac{1}{6}}$

"How can you use estimation and number sense to judge the reasonableness of your answers?"

NOTES

TALK the TALK

Strategies

Write a 1–2 paragraph summary of this lesson. Be sure to address each question.

1. **Describe your basic strategy for evaluating any algebraic expression.**

2. **How are tables helpful when evaluating expressions?**

(1) (1) Take out the letter, & put in the #.
(put parenthesis around the #)
(2) Follow the order of operations to simplify.

(2) Tables keep you organized

Assignment

Write

Explain the difference between an algebraic expression and a linear expression.

Remember

To evaluate an algebraic expression, replace each variable in the expression with a number or numeric expression and then perform all possible mathematical operations.

Practice

Evaluate each algebraic expression.

1. $64 - 9p$ for $p = 4, 9, -3$
2. $-w + 8.5$ for $w = 12, -1.5, 5.3$
3. $46 + (-2k)$ for $k = 3, 23, -2$

Complete each table.

4.

b	$3b + 14$
-5	
-3	
0	
4	

5.

v	1	2	5	-3.25
$6.75 - 6v$				

6.

f	4	8	-12	-1
$\frac{f}{4} + 3f$				

Evaluate each algebraic expression for the given quantity.

7. $-6.2x + 1.4x,\ x = -9.3$
8. $3\frac{1}{2}x - 5\frac{1}{3}x,\ x = \frac{2}{5}$

Stretch

Evaluate each algebraic expression for the given quantity.

1. $-3(2.1x - 7.9)$ for $x = -18.1, -0.3, 14.4$
2. $-9.8t^2 + 20t + 8$ for $t = -2, 0, 3.5$

Review

Rewrite each numeric expression by factoring out (−1).

1. $-7 + 5$
2. $3 + 8$

Determine each quotient.

3. $-8.9 \div -0.1$
4. $-4\frac{1}{3} \div \frac{2}{5}$

Write two unit rates for each given ratio.

5. $\frac{3}{4}$ inch : $\frac{1}{2}$ hour
6. $\frac{4}{5}$ gallon : $\frac{1}{4}$ cup

Mathematics Gymnastics 2

Rewriting Expressions Using the Distributive Property

Math 7 mod 3 Topic 1 L2

WARM UP

Write a numeric expression for the opposite of each given expression.

1. $-7 - 2$

2. $3 - 9$

3. $-3 + 2$

4. $3 - (-7)$

LEARNING GOALS

- Write and use the Distributive Property.
- Apply the Distributive Property to expand expressions with rational coefficients.
- Apply the Distributive Property to factor linear expressions with rational coefficients.

KEY TERMS

- factor
- coefficient
- common factor
- greatest common factor (GCF)

You have used the Distributive Property to expand and factor algebraic expressions with positive numbers. How can you apply the property to all rational numbers?

Getting Started

Where Are They?

Consider the list of linear expressions.

$x + 1$ $\quad$ $2x + 2$ $\quad$ $3x + 3$ $\quad$ $4x + 4$

1. On the empty number line, plot each algebraic expression by estimating its location.

2. Explain your strategy. How did you decide where to plot each expression?

3. What assumptions did you make to plot the expressions? Does everyone's number line look the same? Why or why not?

ACTIVITY 2.1

Algebraic Expressions on the Number Line

To **factor** an expression means to rewrite the expression as a product of factors.

Consider the four expressions plotted in the previous activity. How can you prove that you are correct?

Graham

I can use an example by evaluating all four expressions at the same value of x and plot the values.

Let $x = 4$.

$x + 1 = 4 + 1 = 5$

$2x + 2 = 2(4) + 2 = 10$

$3x + 3 = 3(4) + 3 = 15$

$4x + 4 = 4(4) + 4 = 20$

I can plot the expressions at 5, 10, 15, and 20.

Meaghan

The expressions look similar. I can factor out the coefficient of each expression.

$x + 1$

$2x + 2 = 2(x + 1)$

$3x + 3 = 3(x + 1)$

$4x + 4 = 4(x + 1)$

So, I can plot $x + 1$ and use that expression to plot the other expressions.

1. Use Graham's strategy with a different positive value for x to accurately plot the four expressions.

$x = 2$

$x+1 = 2+1 = 3$

$2x+2 = 2(2)+2 = 4+2 = 6$

$3x+3 = 3(2)+3 = 6+3 = 9$

$4x+4 = 4(2)+4 = 8+4 = 12$

A **coefficient** is a number that is multiplied by a variable in an algebraic expression.

2. Use Graham's strategy with a negative value for x to accurately plot the four expressions. How is your number line different from the number line in Question 1?

$x = -4$

$x+1 = -4+1 = -3$

$2x+2 = 2(-4)+2 = -8+2 = -6$

$3x+3 = 3(-4)+3 = -12+3 = -9$

$4x+4 = 4(-4)+4 = -16+4 = -12$

Often, writing an expression in a different form reveals the structure of the expression. Meaghan saw that each expression could be rewritten as a product of two factors.

Meaghan's expressions

$x + 1$

$2x + 2 = 2(x + 1)$

$3x + 3 = 3(x + 1)$

$4x + 4 = 4(x + 1)$

3. What are the two factors in each of Meaghan's expressions? What is common about the factors of each expression?

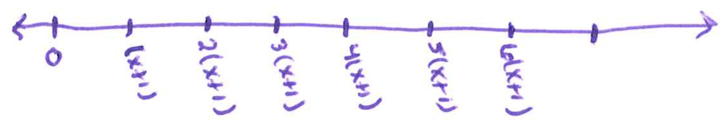
$2(x+1) \rightarrow 2$ and $(x+1)$ $\qquad$ $x+1 \rightarrow 1$ and $(x+1)$

$3(x+1) \rightarrow 3$ and $(x+1)$ $\qquad$ Note: $x+1 = 1(x+1)$

$4(x+1) \rightarrow 4$ and $(x+1)$

Each of them have a factor of $x+1$

4. Use Meaghan's work to accurately plot the four expressions. Explain your strategy.



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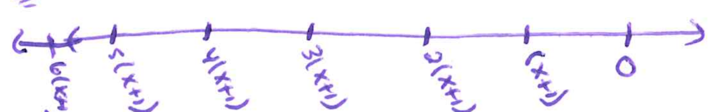


If a variable has no coefficient, the understood coefficient is 1.

5. Meaghan noticed that the expressions formed a sequence. Write and plot the next two terms in the sequence. Explain your strategy.

$5(x+1)$ $\qquad$ The # in front of $(x+1)$

$6(x+1)$ $\qquad$ goes up 1 each time.

6. What property did Meaghan use when she factored out the coefficient of the expressions?

She used the distributive property.

Recall:

Distributive Property

$a(b+c) = a \cdot b + a \cdot c$

ACTIVITY
2.2

Applying the Distributive Property

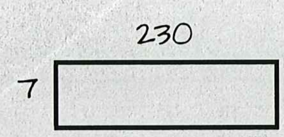
NOTES

Recall that the Distributive Property states that if *a*, *b*, and *c* are any real numbers, then $a(b + c) = ab + ac$. The property also holds if addition is replaced with subtraction, then $a(b - c) = ab - ac$.

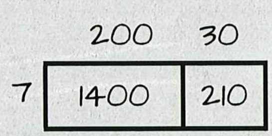
Dominique remembers that the Distributive Property can be modeled with a rectangle. She illustrates with this numeric example.

Dominique

Calculating 230×7 is the same as determining the area of a rectangle by multiplying the length by the width.



But I can also decompose the rectangle into two smaller rectangles and calculate the area of each. I can then add the two areas to get the total.



$1400 + 210 = 1610$

So, $7(230) = 1610$.

1. **Write Dominique's problem in terms of the Distributive Property.**

$7(230) = 7(200 + 30)$

$7(200) + 7(30)$

$1400 + 210 = \boxed{1610}$

You can also use area models with algebraic expressions.

2. Draw a model for each expression, and then rewrite the expression with no parentheses.

a. $6(x + 9)$

$x \quad +9$

6	$6x$	54

$6x+54$

b. $7(2b - 5)$

$2b \quad -5$

7	$14b$	-35

$14b-35$

c. $-2(4a + 1)$

$4a \quad +1$

-2	$-8a$	-2

$-8a-2$

d. $\frac{x + 15}{5} = \frac{1}{5}(x+15)$

$x \quad +15$

$\frac{1}{5}$	$\frac{1}{5}x$	3

$\frac{1}{5}x+3$

3. Use the Distributive Property to rewrite each expression in an equivalent form.

a. $3(4y + 2)$

$3 \cdot 4y + 3 \cdot 2$

$12y+6$

b. $12(x + 3)$

$12 \cdot x + 12 \cdot 3$

$12x+36$

c. $-4a(3b - 5)$

$-4a \cdot 3b \quad -4a \cdot -5$

$-12ab + 20a$

d. $-7(2y - 3x + 9)$

$-7 \cdot 2y \quad -7 \cdot -3x \quad -7 \cdot 9$

$-14y + 21x - 63$

e. $\frac{6m + 12}{-2}$

$\frac{6m}{-2} \quad \frac{12}{-2}$

$-3m-6$

or $-\frac{1}{2}(6m+12)$

$-\frac{1}{2} \cdot 6m \quad -\frac{1}{2} \cdot 12$

$-3m-6$

f. $\frac{22 - 4x}{2}$

$\frac{22}{2} - \frac{4x}{2}$

$11-2x$

or $\frac{1}{2}(22-4x)$

$\frac{1}{2} \cdot 22 \quad \frac{1}{2} \cdot -4x$

$11-2x$

4. Simplify each expression. Show your work.

a. $-6(3x + (-4y))$

$-6 \cdot 3x + -6 \cdot -4y$

$-18x + 24y$

b. $-4(-3x - 8) - 34$

$-4 \cdot -3x \quad -4 \cdot -8 \quad -34$

$12x + 32 - 34$

$12x-2$

Note

$-2(3x \; -4)$

$-2 \cdot 3x \quad -2 \cdot -4$

$-6x+8$

* I like to circle the negatives & then draw the lines. It helps remember the sign.

c. $\dfrac{-7.2 - 6.4x}{-0.8}$

$\dfrac{-7.2}{-0.8}$ $\dfrac{-6.4x}{-0.8}$

$\boxed{9 + 8x}$

d. $\left(-2\frac{1}{2}\right)\left(3\frac{1}{4}\right) + \left(-2\frac{1}{2}\right)\left(-2\frac{1}{4}\right)$

$-2\frac{1}{2}\left(3\frac{1}{4} - 2\frac{1}{4}\right)$ or $-8\frac{1}{8} + 5\frac{5}{8}$

$-2\frac{1}{2}(1)$ $-2\frac{4}{8}$

$-2\frac{1}{2}$

e. $\dfrac{\left(-7\frac{1}{2}\right) + 5y}{2\frac{1}{2}}$

$\dfrac{-7\frac{1}{2}}{2\frac{1}{2}} + \dfrac{5y}{2\frac{1}{2}}$

$\boxed{-3 + 2y}$

5. Evaluate each expression for the given value. Then, use properties to simplify the original expression. Finally, evaluate the simplified expression.

a. $2x(-3x + 7)$ for $x = -1\frac{2}{3}$

$2\left(-1\frac{2}{3}\right)\left[-3\left(-1\frac{2}{3}\right) + 7\right]$

$-3\frac{1}{3}\,[5 + 7]$

$-3\frac{1}{3}(12)$ -40

b. $\dfrac{4.2x - 7}{1.4}$ for $x = 1.26$

$\dfrac{4.2(1.26) - 7}{1.4} = \dfrac{5.292 - 7}{1.4} = \dfrac{-1.708}{1.4} = \boxed{-1.22}$

c. Which form—simplified or not simplified—did you prefer to evaluate? Why?

6. **A student submitted the following quiz. Grade the paper by marking each correct item with a √ or incorrect item with an X. Correct any mistakes.**

Name Alicia Smith

Distributive Property Quiz

a. $2(x + 5) = 2x + 10$

b. $2(3x - 6) = 6x - 6$

c. $-3x(4y - 10) = -12xy + 30$

d. $5x(3x + 2y) = 15x + 10xy$

e. $\frac{15x + 10}{5} = 3x + 2$

f. $\frac{8x - 4}{4} = 2x + 1$

g. $12x + 4 = 3(4x + 1)$

h. $-2x + 8 = -2(x - 4)$

ACTIVITY 2.3

Factoring Linear Expressions

You can use the Distributive Property to expand expressions, as you did in the previous activity, and to factor linear expressions, as Meaghan did. Consider the expression:

$$7(26) + 7(14)$$

Since both 26 and 14 are being multiplied by the same number, 7, the Distributive Property says you can add 26 and 14 together first, and then multiply their sum by 7 just once.

$$7(26) + 7(14) = 7(26 + 14)$$

You have factored the original expression.

The number 7 is a *common factor* of both 7(26) and 7(14).

A **common factor** is a number or an algebraic expression that is a factor of two or more numbers or algebraic expressions.

The **greatest common factor (GCF)** is the largest factor that two or more numbers or terms have in common.

LCM

1. Factor each expression using the Distributive Property.

a. 4(33) − 4(28)

4(33 - 28)
4(5) 20

b. 16(17) + 16(13)

16 (17 + 13)
16 (30)

The Distributive Property can also be used to factor algebraic expressions. For example, the expression 3x + 15 can be written as 3(x) + 3(5), or 3(x + 5). The factor, 3, is the *greatest common factor* to both terms.

When factoring algebraic expressions, you can factor out the greatest common factor from all the terms.

WORKED EXAMPLE

Consider the expression 12x + 42.

The greatest common factor of 12x and 42 is 6. Therefore, you can rewrite the expression as 6(2x + 7).

How can you check to make sure you factored correctly?

It is important to pay attention to negative numbers. When factoring an expression that contains a negative leading coefficient it is preferred to factor out the negative sign.

WORKED EXAMPLE

Consider the expression $-2x + 8$. You can think about the greatest common factor as being the coefficient of -2.

$$-2x + 8 = (-2)x + (-2)(-4)$$
$$= -2(x - 4)$$

2. Rewrite each expression by factoring out the greatest common factor.

① Find the GCF
② divide each term by the GCF.
③ write the answer

GCF = 7

a. $\frac{7x}{7} + \frac{14}{7}$

$\boxed{7(x+2)}$

b. $\frac{9x}{9} - \frac{27}{9}$ GCF = 9

$\boxed{9(x-3)}$

GCF = 5

c. $\frac{10y}{5} - \frac{25}{5}$

$\boxed{5(2y-5)}$

d. $\frac{8n}{4} + \frac{28}{4}$ GCF = 4

$\boxed{4(2n+7)}$

GCF = -3

e. $\frac{-3x}{-3} - \frac{27}{-3}$

$\boxed{-3(x+9)}$

f. $\frac{-6x}{-6} + \frac{30}{-6}$ GCF = -6

$\boxed{-6(x-5)}$

* if the first term is negative - factor out a negative.

So, when you factor out a negative number all the signs will change.

Often, especially in future math courses, you will need to factor out the coefficient of the variable, so that the variable has a coefficient of 1.

3. Rewrite each expression by factoring out the coefficient of the variable.

a. $\frac{10x}{10} - \frac{45}{10}$

$\boxed{10(x-4.5)}$

b. $\frac{-2x}{-2} + \frac{3}{-2}$

$\boxed{-2(x-1\tfrac{1}{2})}$

c. $\frac{-x}{-1} + \frac{4}{-1}$

$\boxed{-1(x-4)}$

d. $\frac{-x}{-1} - \frac{19}{-1}$

$\boxed{-1(x+19)}$

NOTES

4. Rewrite each expression by factoring out the GCF.

GCF = -8 a. $-24x + 16y$

$\frac{-24x}{-8}$ $\frac{16y}{-8}$

$\boxed{-8(3x-2y)}$

b. $-4.4 - 1.21z$ GCF = -1.1

$\frac{-4.4}{-1.1}$ $\frac{1.21z}{-1.1}$

$\boxed{-1.1(4+1.1z)}$

GCF = -3 c. $-27x - 33$

$\frac{-27x}{-3}$ $\frac{33}{-3}$

$\boxed{-3(9x+11)}$

d. $-2x - 9y$ GCF = -1

$\frac{-2x}{-1}$ $\frac{9y}{-1}$

$\boxed{-1(2x+9y) \text{ or } -(2x+9y)}$

GCF = x e. $4x + (-5xy) - 3x$

$\frac{4x}{x}$ $\frac{(-5xy)}{x}$ $\frac{3x}{x}$

$x(4+(-5y)-3)$

$x(1+(-5y))$ or $\boxed{x(1-5y)}$

5. Evaluate each expression for the given value. Then factor the expression and evaluate the factored expression for the given value.

a. $-4x + 16$ for $x = 2\frac{1}{2}$

b. $30x - 140$ for $x = 5.63$

Evaluate:

$-4(2\frac{1}{2})+16 = -10+16 = 6$

$30(5.63)-140$

$168.9-140$

$\boxed{28.9}$

Factor:

$\frac{-4x}{-4}$ $\frac{+16}{-4}$ = $\boxed{-4(x-4)}$

$-4(2\frac{1}{2}-4)$

$-4(-1\frac{1}{2}) = 6$

$\frac{30x}{10}$ $\frac{-140}{10}$

$\boxed{10(3x-14)}$

$10[3(5.63)-14] = 10[16.89-14]$

$10(2.89)$

$\boxed{28.9}$

c. Which form—simplified or not simplified—did you prefer to evaluate? Why?

NOTES

TALK the TALK

Flexible Expressions

As you have seen, you can rewrite expressions by factoring out a GCF or by factoring out the coefficient of the variable. You can also rewrite expressions by factoring out any value. For example, some of the ways $6x + 8$ can be rewritten are provided.

$2(3x + 4)$	$6\left(x + \frac{4}{3}\right)$	$-2(-3x - 4)$
$-6\left(-x - \frac{4}{3}\right)$	$\frac{1}{2}(12x + 16)$	$-\frac{1}{2}(-12x - 16)$

Rewrite each expression in as many ways as you can by factoring the same value from each term.

1. $4x - 12$

2. $-3x + 15$

3. $10 - 20y$

4. $-8y + 9$

Assignment

Write

Match each term to the correct example.

1. factor
2. coefficient
3. common factor

a. the 6 in $6(x) + 6(3)$
b. $-6x - 18 = -6(x + 3)$
c. the 4 in $4x + 3$

Remember

The Distributive Property states that if a, b, and c are any real numbers, then $a(b + c) = ab + ac$.

The Distributive Property makes it possible to write numeric and algebraic expressions in equivalent forms by expanding and factoring expressions.

Practice

Use the Distributive Property to rewrite each expression in its equivalent form.

1. $4(x + 3)$
2. $-7(4 - y)$
3. $6(3x + 5y - 4)$
4. $\frac{9a - 3}{3}$
5. $\frac{0.4(0.3m + 0.6n)}{1.2}$
6. $-9\frac{2}{3}(-2\frac{1}{4}a + b + 8\frac{1}{4})$

Rewrite each linear expression by factoring out the greatest common factor.

7. $64x + 24$
8. $-5y - 35$
9. $36 - 8z$
10. $54n - 81$

Rewrite each linear expression by factoring out the coefficient of the variable.

11. $-2x + 5$
12. $3x - 8$
13. $\frac{-1}{2}x + 6$
14. $-x - 10$

Stretch

1. Jack decides to grow and sell bean plants. Let p represent the number of plants he will grow and sell. After considering his expenses, the expression $-3p(p - 10) - 6p(p - 10)$ represents his profit.
   a. Rewrite and simplify the profit expression by factoring out the greatest common factor.
   b. Rewrite the expression in simplest form with no parentheses.

Review

Evaluate each expression for the given value.

1. $-20a - 65$ for $a = 2.7$

2. $-6x + 52$ for $x = 1\frac{1}{6}$

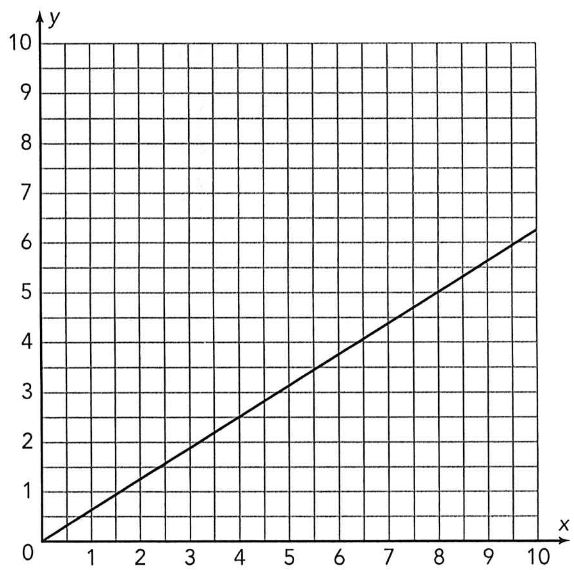
Determine each product.

3. $(-3.472)(0.89)$

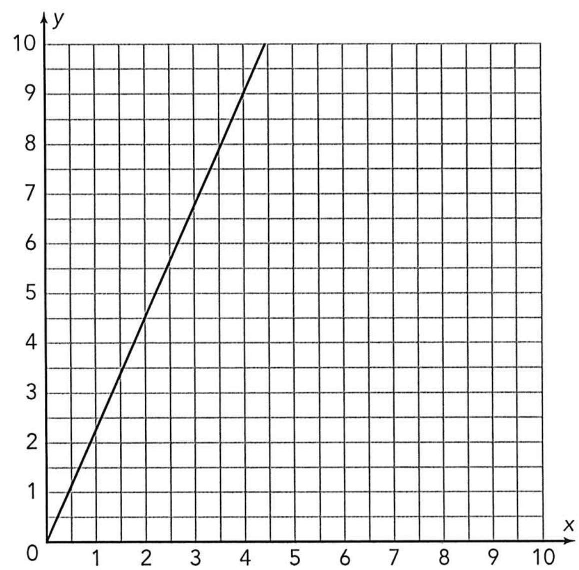
4. $\left(-2\frac{7}{8}\right)\left(-4\frac{4}{5}\right)$

Identify the constant of proportionality in each graph and use it to write an equation in the form $y = kx$.

5.



6.



All My Xs

Combining Like Terms

3

Math 7 mod 3 Topic 1 L3

WARM UP

Simplify each expression using the Order of Operations.

1. $10 + 8 \div 2(4 + 3)$

2. $21 \div (1)(3) + 0 - 14$

3. $18.2 + 6.1(5) - 3.5$

4. $\frac{3}{4} - \frac{4}{5}(2 - 10)$

LEARNING GOALS

- Model adding and subtracting linear expressions on the number line.
- Determine sums and differences of linear algebraic expressions with rational coefficients.
- Combine like terms to simplify linear expressions and determine sums and differences.
- Write and evaluate algebraic expressions to model situations.
- Rewrite expressions in different forms in context to shed light on how quantities in a problem are related.

KEY TERMS

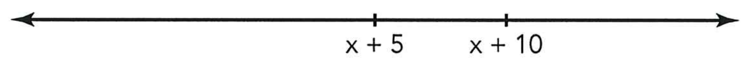
- like terms

You have evaluated algebraic expressions and determined equivalent expressions using the Distributive Property. How can you add and subtract algebraic expressions?

Getting Started

Find X

The number line shows the locations of two values, represented by linear algebraic expressions.



1. Draw and label a tick mark on the number line to locate each expression given. Explain the reasoning you used to determine the location.

a. x

b. $x + 15$

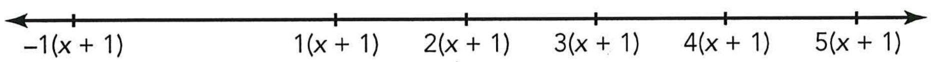
c. $x - 5$

ACTIVITY
3.1

Combining Like Terms in Linear Expressions

NOTES

The number line shows the locations of six values, represented by linear expressions.



1. Determine the distance between 3(x + 1) and 1(x + 1) on the number line. Describe the reasoning you used to determine the distance.

2(x+1) It's two marks & each mark is x+1.

Like terms are parts of an algebraic expression that have the same variable expression. For example, in the expression $5(x + 2) + 3 - 2(x + 2)$, the terms $5(x + 2)$ and $2(x + 2)$ are like terms. The coefficients, 5 and 2, are different, but the variable expression is the same $(x + 2)$.

NOTES

When you operate with like terms, you can combine like terms before doing other operations.

WORKED EXAMPLE

Rewrite the expression $5(x + 2) + 3 - 2(x + 2)$ by combining like terms.

$3 + 5(x + 2) - 2(x + 2)$	You can rewrite the expression using the Commutative Property of Addition.
$3 + 3(x + 2)$	Combine the like terms.

2. Use the worked example to answer each question.

a. How is the Commutative Property used to rewrite the expression?

$5(x+2) + 3 = 3 + 5(x+2)$

b. How are the like terms in the expression combined?

$5(x+2) - 2(x+2) = \boxed{3(x+2)}$

$5 - 2 = 3$

3. Consider the expressions $3(x + 1)$ and $1(x + 1)$ from Question 1.

a. Explain how these two expressions can be "like terms."

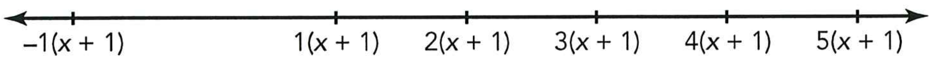
They both have $(x+1)$

b. How did you combine like terms to determine the distance between the expressions? Use the number line to explain your reasoning.

$3(x+1)$ on the number line means 3 groups of $(x+1)$, There are 2 groups of $(x+1)$ between $3(x+1)$ and $(x+1)$. So, $3(x+1) - 1(x+1) = 2(x+1)$

4. **First, use the number line to determine the distance between the given expressions. Then write an expression and show how combining like terms produces the same result.**

The number line given is separated into intervals of $(x + 1)$.



a. **Determine the distance between $5(x + 1)$ and $2(x + 1)$.**

$5(x+1) - 2(x+1) = \boxed{3(x+1)}$

b. **Determine the distance between $1(x + 1)$ and $-1(x + 1)$.**

$1(x+1) - -1(x+1)$

$1(x+1) + 1(x+1) = \boxed{2(x+1)}$

5. **Explain why Kaitlyn's reasoning is incorrect. Then rewrite the expression correctly.**

Kaitlyn

$7 - 5(x + 1)$ is the same as $2(x + 1)$, because $7 - 5 = 2$.

She is not subtracting like terms.
7 doesn't have an (x+1)

6. **Simplify each expression by combining like terms.**

a. $5(x + 2) + 2(x + 2)$

$7(x+2)$

b. $5(x + 2) - 2(x + 2)$

$3(x+2)$

ACTIVITY 3.2 Combining Like Terms with Decimal and Fractional Coefficients

You can combine like terms to determine prices with discounts and with sales tax.

For example, suppose a new toy that is regularly priced at \$26.99 is on sale for $\frac{3}{4}$ off.

1. Write an expression to represent the price of the toy, *p*, minus $\frac{3}{4}$ of the price. Then, combine like terms to simplify the expression.

$p - \frac{3}{4}p = \frac{1}{4}p$ ☆ $1 - \frac{3}{4} = \frac{1}{4}$

2. Explain what the simplified expression means in terms of the original price of the toy.

The simplified expression $\frac{1}{4}p$, means that the discounted price is $\frac{1}{4}$ of the original Price.

A new shirt costs \$18.99. The sales tax is 5%.

3. Write an expression to represent the cost of the shirt, *s*, plus 5% of the cost. Then, combine like terms to simplify the expression.

☆ 1.05 = 105%

100% of the item plus 5% tax.

$s + .05s = 1.05s$

4. Explain what the simplified expression means in terms of the original cost of the shirt.

The simplified expression, 1.05s, means that the price with tax is 1.05 times the original price.

5. Write and simplify an algebraic expression to represent each situation.

a. An 18% tip is given for a meal. What expression represents the total cost with tip?

let m = the cost of the meal

total cost = $1.18m$

b. A pair of shoes is advertised as $\frac{1}{4}$ off. What expression represents the total cost after the discount?

let s = the original cost of the shoes

Total cost = $\frac{3}{4}s$

* $\frac{1}{4}$ off = pay $\frac{3}{4}$

c. A new bike is discounted 35%. What expression represents the total cost?

let b = the original price of the bike.

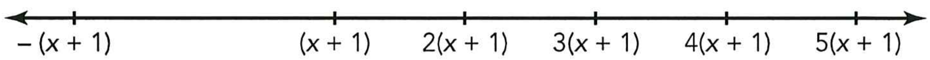
Total cost = $.65b$.

* 35% off = pay 65%

ACTIVITY 3.3

Adding the Opposite to Subtract

Sasha was able to combine like terms to determine the distance between $3(x + 1)$ and $(x + 1)$ on the number line. She knew that the distance was $2(x + 1)$.



But when she showed her work using the Distributive Property, she got the wrong answer.

Sasha

$3(x + 1) - (x + 1)$

$3x + 3 - x + 1$

$3 + 3x - x + 1$

$3 + 2x + 1$

$2x + 4 = 2(x + 2)$

1. Julian said that Sasha made a mistake when subtracting $(x + 1)$. He said that subtracting $(x + 1)$ is the same as adding the opposite of $(x + 1)$.

a. What is the opposite of $(x + 1)$? Write your answer without parentheses.

$-(x+1) = -x-1$

b. Show Sasha how adding the opposite produces the correct answer.

$3(x+1) + -x-1 =$

$3x + 3 - x - 1$

$\boxed{2x+2}$

$3x - x = 2x$

$3 - 1 = 2$

Simplify each expression. Use the Order of Operations.

2. $30x - 140 - 1(x - 4)$

$30x - 140 - x + 4$

$\boxed{29x - 136}$

3. $10 - 5(-2r - 13) - 7r$

$10 + 10r + 65 - 7r$

$\boxed{3r + 75}$

4. $-4x + 5(2x - y) - 3y$

$-4x - 10x + 5y - 3y$

$\boxed{-14x + 2y}$

5. $7.6p - 3.2(3.1p - 2.4)$

$7.6p - 9.92p + 7.68$

$\boxed{-2.32p + 7.68}$

6. $3\frac{2}{3}p - 1\frac{3}{4}(4p - 2\frac{1}{7})$

$3\frac{2}{3}p - 7p + 4\frac{3}{4}$

$\boxed{-3\frac{1}{3}p + 4\frac{3}{4}}$

2*x* and 2*y* are not like terms because they don't have the same variable.

NOTES

TALK the TALK

Business Extras

Katie is starting a limousine rental company. As part of her research, Katie discovers that she must charge a 7% sales tax to her customers in addition to her rental fees.

1. **Write an algebraic expression that represents how much tax Katie should collect for any amount of rental fee.**

Katie also discovers that most limousine rental companies collect a flat gratuity from customers in addition to the rental fee. Katie decides to collect a gratuity of $35 from her customers.

2. **Write an expression that represents the total amount of additional money to be collected for tax and gratuity.**

3. **Write an expression that represents the total cost of any rental.**

4. **Use one of your expressions to calculate the amount of tax and gratuity Katie should collect if the rental fee is $220.**

5. **Use one of your expressions to calculate the total cost of a rental if the rental fee is $365.**

Assignment

Write

Write a definition of *like terms* in your own words. Use an example to illustrate your definition.

Remember

Use the Order of Operations to simplify expressions.

1. Simplify expressions in parentheses.
2. Simplify powers.
3. Multiply and divide from left to right.
4. Add and subtract from left to right.

Practice

Simplify each expression by combining like terms.

1. $6x + 4x$
2. $-5y + 2y$
3. $-3m - 8(m + 1)$
4. $-8(r - 2) + 6(r - 2)$
5. $9m - 7m + 13$
6. $4a + 8b$
7. $2(x + 3) + 4(x - 3)$
8. $6 - 2(3x + 4)$
9. $4.5x + 2.5(x - 4)$
10. $3.1 - 4.9(x + 1)$

11. Identify the expression or expressions equivalent to $-5 + 4(-2x + 6) - 3x$.
   a. $-8x + 19 - 3x$
   b. $5x - 29$
   c. $-11x + 19$
   d. $-5x - 19$

Stretch

Sonya is renting a car. She pays a fee of $50 for the rental plus $20 each day she has the car. Suppose she pays a total of $130. For how many days did she rent the car?

Review

Rewrite each expression using the Distributive Property.

1. $5x + 11x$
2. $-2(x - 5)$

Determine each product or quotient.

3. $-\frac{2}{3} \times \frac{1}{5}$
4. $-\frac{1}{8} \div 4$

Solve for the unknown in each proportion. Round to the nearest tenth.

5. $\frac{4.1}{42} = \frac{x}{12}$
6. $\frac{9.7}{1.4} = \frac{5.2}{y}$